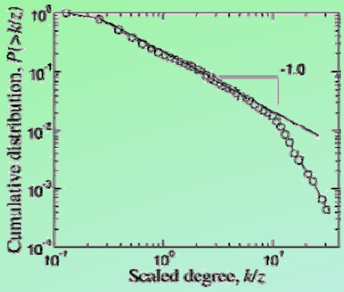
Embedding Time in Complex Networks

Massimiliano Zanin, Miguel Cea and Samuel Cristóbal

Innaxis Foundation & Research Institute, Spain

The problem

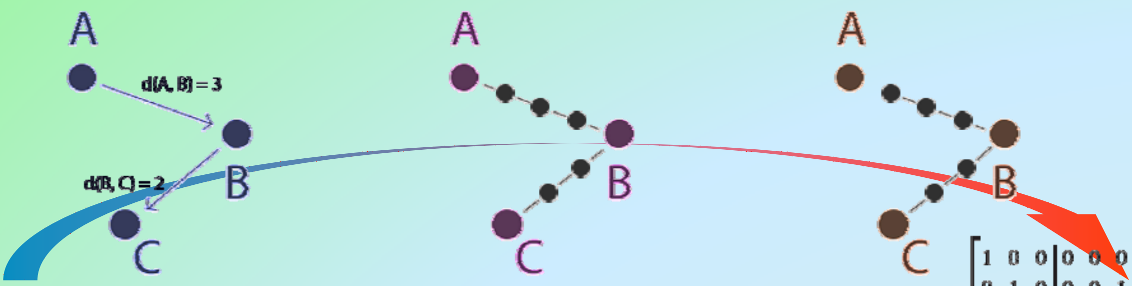
When studying complex systems whose dynamics are embedded in an underlying network of interactions, it is common to define the architecture of such networks as a static structure that characterizes which node is connected to which other. In this sense, not much attention has been paid to the possibility of the network structure (i.e. its adjacency matrix) to be dynamically modified by external mechanisms. However, real networks are indeed dynamically modified in many cases: for instance the time restrictions (schedule) in transportation systems.



Degree distribution of the worldwide aeronautical network.

If the time is not considered, and no scheduling information is included, results can be unrealistic.

From Complex to Scheduled Networks

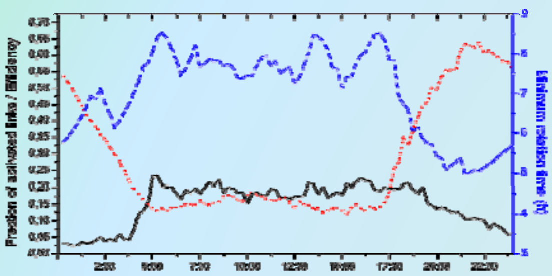


$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

To include the limitations imposed by time, we define a dynamical network as a collection of *primary nodes* - which correspond to the original nodes of the graph - and *secondary nodes*, that are used to delay movements according to the scheduling information.

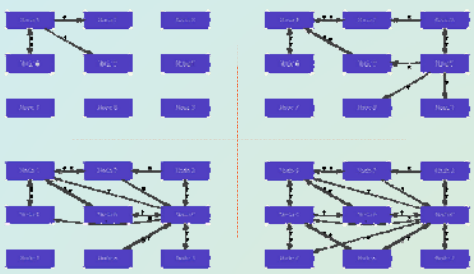
$$dA = \left[\begin{array}{c|c} P & R \\ \hline act & T \end{array} \right] = \left[\begin{array}{ccc|ccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Applying Scheduled Network to the Air Transportation Networks



Example of some dynamical metrics calculated for the air transportation network: efficiency (relation between the minimal and the real flight time) and minimal rotation time.

Scheduled Networks can be also used to realistically simulate the evolution of a network, based on the number of passengers and on the fitness of each node.



M. Zanin, L. Lacasa and M. Cea, Dynamics in Scheduled Networks, *Chaos*, **19**, 023111 (2009).

S. Boccaletti, V. Latora, Y. Moreno, M. Chavez and D.-U. Hwang, Complex networks : Structure and dynamics, *Physics Reports*, **424**, 4-5 (2006) p. 175--308.

L. da F. Costa, O. N. Oliveira, G. Travieso, F. A. Rodrigues, P. R. V. Boas, L. Antiqueira, M. P. Viana and L. E. Correa da Rocha, Analyzing and Modeling Real-World Phenomena with Complex Networks: A Survey of Applications, *arXiv:0711.3199v2 [physics.soc-ph]*, (2007).