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Networks of springs: a dynamical approach

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Abstract

In the present work we study the propagation of perturbations through networks of springs which are spatially distributed. We show that the topological properties of the network are related with the dissipation of the energy within the system. These results are of potential application to the design of more efficient dumping systems.

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1. Introduction

The field of complex networks has attracted the attention of many researchers in recent years, partly because of the easiness of applying complex networks theory to many different real-world problems. In particular, many interesting results have been obtained in social networks, communication and traffic control, or system security and failure analysis. An extensive review can be found in Refs. [1, 2].

Most of those systems are based in dimensionless graphs, where the spatial position of each node is not significant when analyzing real data. In this work, our approach may be seen as the opposite: we study the dynamical properties of a spatial, 2-dimensional real physical systems, using concepts that can be found in classical complex networks literature.

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2. The physical system

The system analyzed in this work is drawn in Fig. 1. In this network, nodes represent masses (of unitary weight), connected to each other with classical springs that follow the differential equation:

$$-kx - \beta\frac{dx}{dt} = m\frac{d^2x}{dt^2} \tag{1}$$

where k is the spring constant (or force constant), and β is the damping coefficient. Movements of nodes are restricted to the XY plane, and masses in the left and right boundary are fixed. Finally, different perturbations are applied to the lower part of the system, and resulting movements are recorded in the upper row. In this way, we treat the spring network as a "black box", where an input perturbation is applied (first row of the network) and an output is observed (last row of the network).

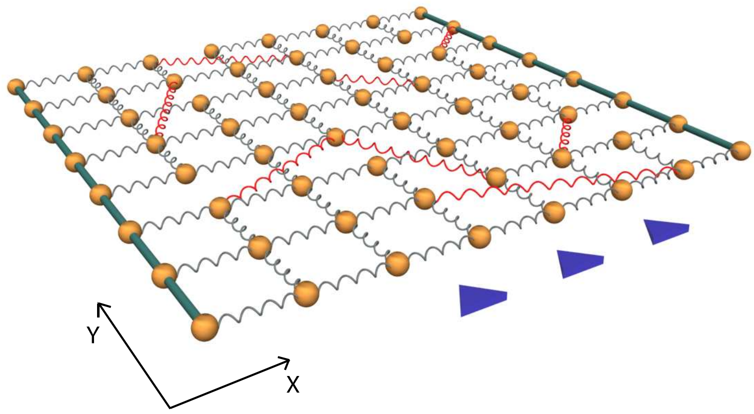

Figure 1: 3D graphical representation of the 8x8 springs network. Lateral nodes are fixed and perturbations are applied to the first row (y=0).

When links between nodes are arranged in a regular structure (i.e., each node connected with its four nearest neighbors) the system behaves as a classical spring, i.e., propagates a damped wave with a well defined frequency. Nevertheless, if we introduce *rewirings* in the node connections with some probability α (red springs in Fig. 2), and apply a force at the bottom row of the network, the propagation of this perturbation towards the upper row suffers important nonlinear effects.

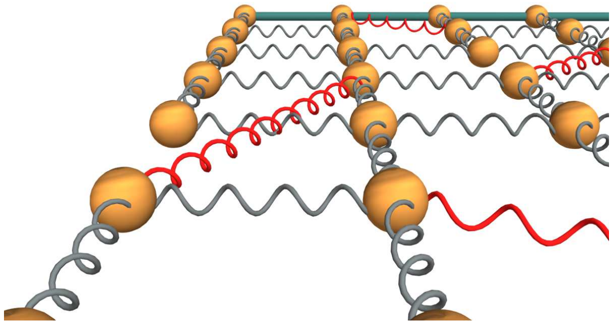

Figure 2: Detail of the network plotted in Fig. 1; observe the red springs, that have been randomly rewired, and that generate a nonlinear long-range effect in the transmission of the perturbation.

3. Numerical results

One of the aims of this work is to find network structures that reduce the last-row oscillations ("output") when a perturbation is applied, even when damping is not considered. In this way we would have increased the efficiency of the dumping system.

A (single) initial perturbation is applied at the first row of nodes in the Y direction and propagates from bottom to top. In Fig. 3, the mean displacement of nodes in the upper row is plotted as a function of the rewiring probability α, for a fixed instantaneous perturbation in the lower row.

The rewiring effectively reduces the output energy, and this is because each node transforms the linear movement of nodes in the regular network into a circular oscillation around their steady point. Figure 3 shows the probability distribution function of the position of each node when a rewiring of $\alpha = 0.5$ is applied.

The effect of the rewiring can be regarded as the sum of two different contribution:

1. Since links (i.e., springs) can be arranged in any direction, and not only in the X or Y direction, the initial linear *down-up* movement is deflected and rotated in any direction, with a new X component that reduces the original Y displacement in the Y direction.

2. Rewiring introduces long-range connections, that break the symmetry and the synchronization of the system.

In order to check the importance of the last point, i.e. the long range effect, in Fig. 4 the mean output displacement is shown as a function of the

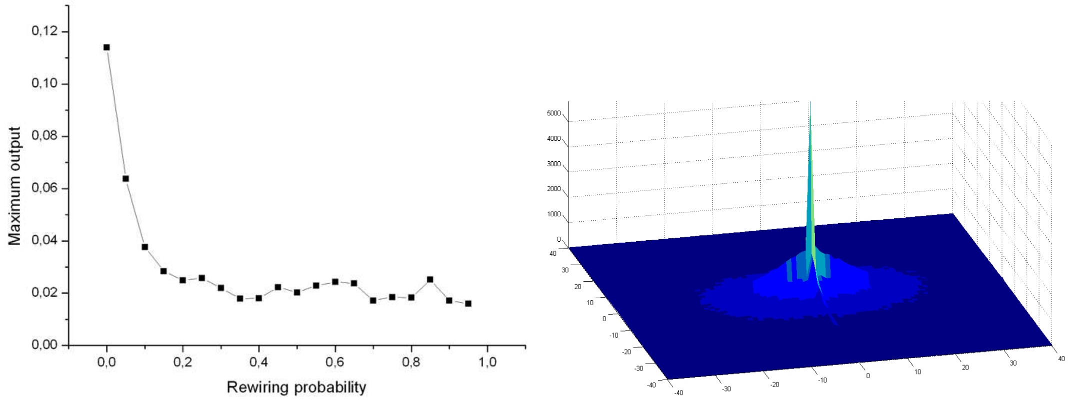

Figure 3: (Left) Mean output amplitude as a function of the rewiring parameter α. Thanks to the rewiring, the output amplitude is reduced, and this is due to a circular movement of nodes around their steady point. (Right) Probability distribution of the position of each node.

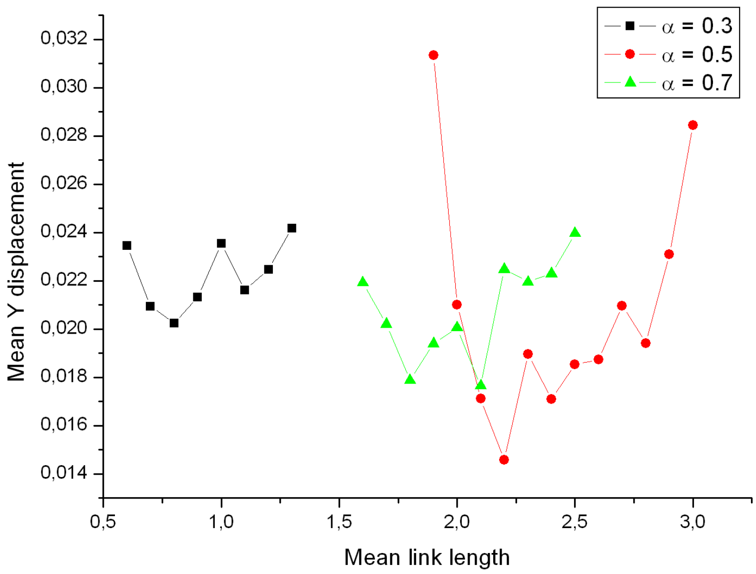

Figure 4: Mean output amplitude as a function of the mean length of the links.

mean links length for fixed rewiring probabilities of $\alpha = 0.3$, $\alpha = 0.5$ and $\alpha = 0.7$. Clearly an optimum solution can be found: for example, when $\alpha = 0.5$, a mean value of approx. 2.2 appears to be the best mean link length, when the distance between two neighbor items is normalized to the unit.

3.. 1 Periodic perturbation

Spring-based system have been widely studied in the past to represent harmonic systems in the specially interesting case of periodic perturbations: the collapse of the old Tacoma Narrows bridge in Seattle is a well-known example of the practical interest in this field.

With the aim of checking the oscillatory properties of the system, a cosine force $F(t) = F_0 cos(\omega t)$ has been applied to the lowest row of the network, and a damping force ($\beta = 0.1$) has been introduced in order to prevent the results to resonate to the infinity.

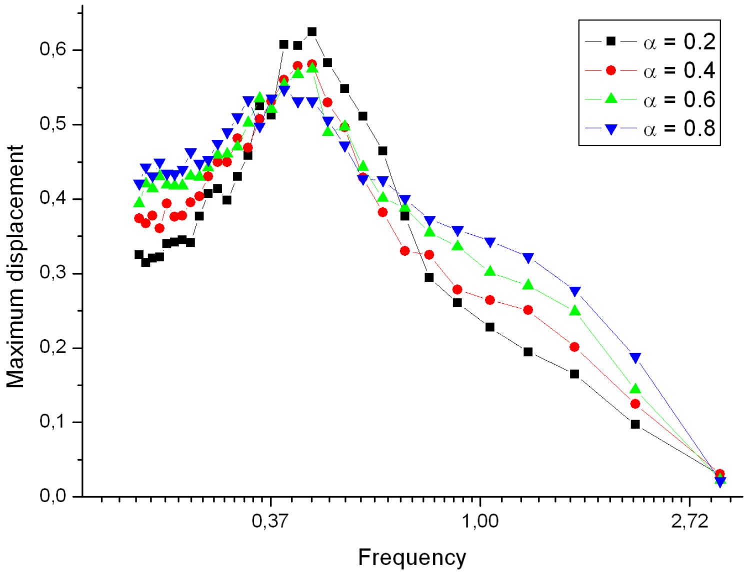

Figure 5: Output amplitude as a function of the frequency of the sinusoidal perturbation applied to the system.

In Fig. 5 we plot the maximum output displacement (that of the upper row) for different rewiring probabilities α. What we observe is the appearance of a resonance in the system, as indicated by a maximum in the displacement distribution.

4. Conclusions

Despite complex networks have been used as a tool for studying of many real systems, only little attention has been devoted to spatial mechanical constructions. In this work, a network made of springs is analyzed, with especial attention for the effects of rewiring; results obtained show that such systems present better perturbation dumping and less sensitivity to periodic forces.

References

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