MEASURING AIRCRAFT FLOWS COMPLEXITY

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Abstract

This contribution studies the problem of measuring the subjective perception of complexity created by non regular intersecting flows of aircraft. By constructing networks connecting aircraft and following their evolution, a Spatial Complexity metric is estimated: comparisons between this metric and a standard one are reported for virtual traffic scenarios.

1 Introduction

Although airspace complexity is an important research field inside the aeronautical world, especially within the actual need for a higher capacity and improved security, little effort has been devoted to measure the subjective perception of complexity introduced by non regular intersecting flows of aircraft. In other words, the assumption behind this contribution is that a situation with aircraft flying following two perpendicular routes is creating less workload than a scenario with the same number of aircraft, but moving in random, non-trivial trajectories (for example, in a 4D SESAR scenario). The standard approach to estimate the complexity of a sector includes two main contributions: (i) the traffic density, as the number of flights crossing a sector in a given time, and (ii) the traffic complexity [1]. The second part is usually approximated by some metrics like minimum distance between aircraft, number of predicted conflicts, or number of intersecting flight paths. An example of a metric which embraces both aspects is the *Dynamic Density*, developed in 1998 by the United States National Aeronautics and Space Administration [2].

Those last metrics fail to fully account for the complexity of the flows created by airplanes, at least to account for the subjective complexity defined at the beginning of this contribution. For example, it is not difficult to imagine a situation with several flights crossing a sector, none of them in conflict with others, but in a configuration which requires the continuous attention of the controller to forecast their future positions. More generally speaking, the aim of this paper is to highlight the importance of this kind of complexity, and to develop a first metric to measure the geometrical complexity of aircraft flows: using a fluid dynamics metaphor, to distinguish laminar from chaotic scenarios.

2 Flows complexity

The metric proposed in this contribution is built around the concept of spatial complexity, introduced by the author within the field of Complex Networks [3]. Complex Networks are generic mathematical objects which have been extensively used to model interactions between elements of a system [4]. Those elements are represented by nodes, and relations between them by links: some examples span from social networks, technological networks, power distribution networks, up to yeast networks, only to cite a few [5]. Thanks to Complex Networks, structures and dynamics of so heterogeneous sources can be easily expressed in terms of standard metrics, and commons characteristics have been found in many natural and man-made systems [6]. In the following, a short description of the spatial complexity is given, and how this measure can be applied to sector complexity is explained.

2.1 An overview of spatial complexity

Approaches to calculate the disorder of a complex network have focused on the analysis of degree heterogeneity, that is, on the existence of a few highly connected nodes, usually called Hubs. This heterogeneity is strongly related with the resilience of the network to random or targeted attacks, and therefore has been widely applied to study critical infrastructures [7]. The problem presented here is different, as intuitively the position of nodes (each node will represent an aircraft) is of utmost importance in understand the complexity of the system. Most of the works dealing with spatial networks usually disregard the spatial information; nevertheless, when considering aircraft's trajectories, this information has to be taken into account, as two airplanes flying far away do not represent the same situation as two aircraft passing a hundred meters from each other.

In Fig. 1 three different spatial networks are represented. The second and third networks are evolutions of the preceding one, where nodes have been moved according to the arrows (note that the structure of connections has not been changed). Standard metrics are not useful to understand this evolution, as they are not modified when the movement is applied; for instance, the mean distance between two nodes is constant, the number of connections of each node has not changed at all, nor other structural metrics like the number of triangles in the network (i.e., the *clustering*). In spite of the above, clearly the three graphs are not equivalent, and the reader may agree that the third one is more *complex* (or more disordered) than the first.

In Ref. [3] this problem is solved by measuring the quantity of information needed by an agent to go from one node to another node through the path of minimal length. Suppose that the agent has only local information about the topology of the network, thus is moving at each step to the node which appears closer to the destination: the result in a disorder graph will be a path longer than the optimal one. The difference between both paths is therefore measured as the quantity of information that the agent would need, from an external *supervisor*, to update its representation of the network, and be able to find the shortest path.

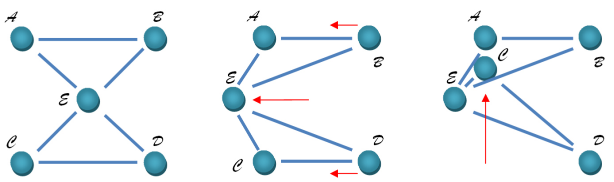

Fig. 1 Example Of Three Spatial Networks With Very Different Complexities.

The reader may check this idea with an example using networks in Fig. 1. Suppose that the agent wants to go from node D to node A, using local information only. In the simple network of the left, the agent (standing in node D), would see two connections: one to node C, and the second to node E; as this last node is closer to the destination (just looking at the Euclidean distance, not at the topology), the agent would move here. In the next step, the destination is on sight, so the trip is completed. Note that, in this ordered network, the chosen path has been also the shortest possible. If we make the same process for the network on the right, results are completely different. At the beginning, the agent will still see connections to nodes C and E, but now the closer to the destination is C; the resulting path will be C, E and A, which is longer than the optimal one.

An interesting feature is that this spatial complexity can be normalized according to the size of the network and to the number of connections in it, which allows comparisons between very heterogeneous graphs. In order to perform such normalization, it is necessary to create multiple reference graphs. Those new networks are created by leaving the nodes of the network to be normalized in the same position, and by creating connections between them at random - the same number of links that were present in the original graph. In other words, we are applying a process of *random rewiring*: the mean value of the complexities of these networks are a reference to understand if we are facing a ordered or disordered system. For example, if we compute the spatial complex-

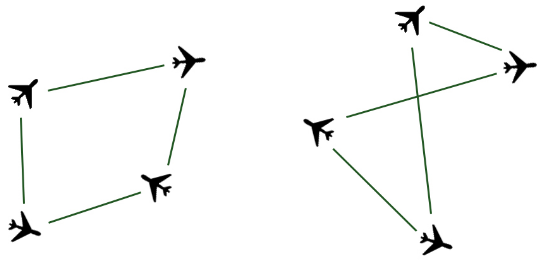

Fig. 2 Calculation Of The Spatial Complexity.

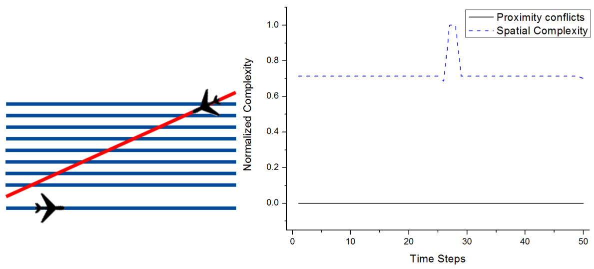

Fig. 3 First Sector Complexity Example.

ity for a given configuration of nodes, and we get a value higher than the one obtained with the normalization, we can infer that our system is more disordered that what would be expected in a random situation - which, in turn, can be explained by the presence of some non-trivial structure inside the topology of nodes.

2.2 Application to sector complexity

After this introduction about how to calculate the spatial complexity of a network, here is introduced how to apply this metric to the problem of airspace complexity. Two steps are needed (see Fig. 2): (i) creating a neutral network at time t, and (ii) calculating the Spatial Complexity of the same network at time $t + \delta t$.

In the first step, an initial network (with a given number of connections) is created using the position of aircraft. This network should have a zero complexity - i.e. as regular as possible; this can be accomplished by casting six links from each aircraft in the six spatial directions, up to the closer aircraft. In Fig. 2 Left a simple bi-dimensional example is shown: each aircraft is connected to the one on its left - top - right - bottom part.

As time goes by, aircraft update their positions: as links are attached to nodes, the network previously created is also changed. The claim of this contribution is that the Spatial Complexity of the new network (after being normalized) is related with the complexity of aircraft trajectories inside the sector.

As can be inferred from Fig. 2, in order to obtain a Spatial Complexity greater than zero the network must be *twisted*: in other words, aircraft's trajectories should cross and point to opposite directions. On the other side, that is, in a situation where many aircraft are flying in parallel (or almost parallel) routes, the network at time $t + \delta t$ will be similar to the initial network: as overall translation is neglected, the resulting complexity is null.

From all the above, it is clear that the proposed complexity metric refers to the structure of flight paths. Of course, this is just one of the possible complexities that an air traffic controller has to manage: for example, although two flights may have parallel routes, they can be in a proximity conflict; or both flights may be re-routed to dodge a bad weather area; or one of them may have declared emergency, and is performing a drift-down. Therefore this kind of metric should be seen as one ingredient of a higher level complexity mix.

3 Analyzing some examples

In order to better understand the characteristics of a complexity metric based on Spatial Complexity, a couple of virtual examples are reported.

The first of them is drawn in Fig. 3. In this example, there is a regular movement of 9 aircraft going straight from the left to the right side of the sector, following the 9 parallel and horizontal lines (note that just one of these aircraft is represented), while at the same time a tenth plane is crossing from right to left, top to bottom, intersecting the other trajectories. In sake of simplicity, we suppose all movements in a 2D plane, although all calculations can be extended to any 3D space. If distance between aircraft is high enough, there will be no conflicts: therefore

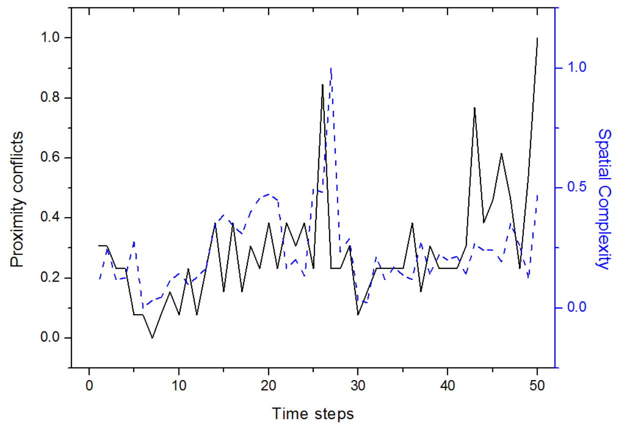

Fig. 4 Second Sector Complexity Example.

any metric based on the number of such events will return a zero complexity - see the solid black line of Fig. 3 Bottom. On the other side, the Spatial Complexity senses the changes in the network structure all over the time, and especially in the middle part (when flights really cross) - dashed blue line in Fig. 3 Right.

As a second example, 40 random flights (that is, entering and exiting the airspace sector from random points) have been created, in order to simulate a highly congested airspace: the evolution of both metrics - a standard one based on the number of forecasted conflicts, and the Spatial Complexity - are shown in Fig. 4. It is interesting to note as the global trend of both series is similar (see the peaks around $t = 27$), although they differ at some points (for example, at $t = 20$ and $t = 45$). Another example is shown in Fig. 5. Other 40 random flights are simulated, but in this case the air sector is ten times bigger than the previous case: therefore proximity alerts are not so frequent, and cannot be used as a metric of complexity.

Summing up, in this scenarios it can be seen that the Spatial Complexity is indeed sensing some kind of complexity of the traffic inside a sector. The complexity which is measured is also different from standard metrics actually used, as is related with the workload generated by non-trivial intersecting trajectories.

4 Computational cost

From Section 2, it may be expected that the computational cost of calculating a complexity measure based on Spatial Complexity is much higher than other standard metrics, like for example the number of aircraft in a sector, or the number of conflicts. Therefore, the computational cost of calculating one step of the algorithm, i.e. the evolution from time t to $t + \delta t$, has been estimated as a function of the number of aircraft in the sector; results are represented in Fig. 6. Simulations have been performed in *Matlab R2008*, and ran in a *Intel Core Duo 2* at $1.67 GHz$

The more aircraft are crossing the airspace, the higher is the time needed to calculate its complexity - the computational complexity has the form of $\Theta(n^2)$. Nevertheless, even for scenarios with 40 airplanes, the calculation does not exceed 0.5 seconds: this solution is suitable to be imple-

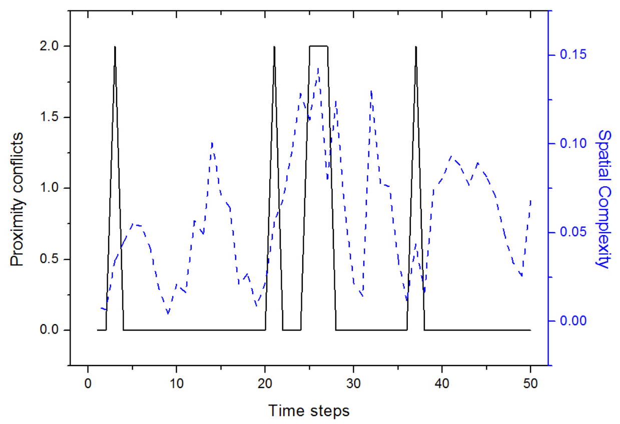

Fig. 5 Third Sector Complexity Example.

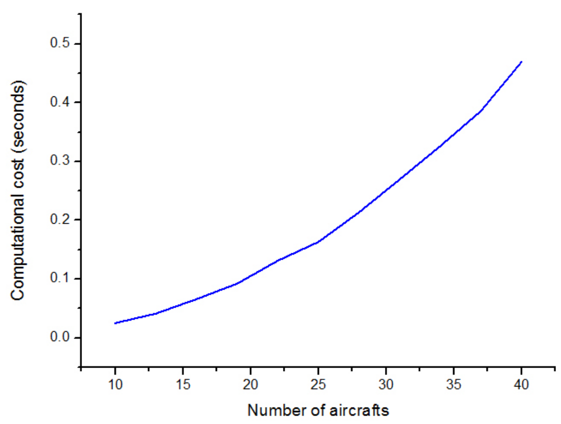

Fig. 6 Computational Cost of the Algorithm.

mented in real time applications. Moreover, the proposed algorithm can easily be adapted for parallel computation, as the most time-consuming part is the normalization: for example, n random networks can be generated in n independent processors, instead of having only one computer calculating sequentially the n different networks.

5 Discussion

In this contribution a new type of sector complexity is introduced, namely the geometrical complexity of aircraft flows: with it, a first approximation is proposed, i.e. the Spatial Complexity. Through several virtual examples, it has been shown that this metric is coherent with more classical complexity metrics; but, at the same time, it can identify chaotic aircraft flows, and therefore add a new dimension in forecasting the controller workload. This seems specially interesting within the future *4D Trajectories* operations, where aircraft will be no more routed through ordered airways, but can cross randomly the airspace. Moreover, this metric is an example of cross-disciplinary contribution: an example of a concept developed in the physical field, and applied to an ATM problem.

Next developments will be focused on validation processes, with real data about the workload of controllers in real situations, as well as better integration with standard complexity measures. Moreover, decision making strategies for aircraft collisions avoidance based on the proposed definition of complexity will be explored.

6 Copyright Statement

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