Embedding Time in Complex Networks

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The understanding of the topology and dynamics of Complex Networks has been a cornerstone in the study of many real and virtual systems which exhibit complex interactions between their elements[1,2].

At the same time, less efforts have been focused in how to embed time inside a Complex Network. If data about the evolution of the network was available, the solution has usually been to track the evolution of some topological metric in consecutive time windows; on the other side, other authors have studied how the internal dynamics of the system can affect the structure: what is called an adaptive complex network[3].

Here we want to propose a different approach which is especially useful when analyzing transportation networks. In such systems, time can be found in two forms: (i) a *scheduling*, which defines when a given link is activated, and (ii) a travel time, as long as movements inside the network are not instantaneous. To handle such situations, we developed an extension of Complex Networks which includes the previous characteristics inside the Adjacency Matrix: what we call a Scheduled Network[4]. Besides the *main nodes*, which are the classical nodes of the system, a certain number of *secondary nodes* are created for each link, to account for the travel time. At the same time, a part of the whole Adjacency Matrix is defined as time-dependent, to dynamically activate or deactivate connections. An example can be seen in Fig. 1. In the Left image is shown a (static) Complex Network with two links, each one defined with a length; in the Center, the corresponding Scheduled Network is constructed: note the 5 *secondary nodes* added to represent the travel time; in the Right, the same network is plotted, but with all links deactivated.

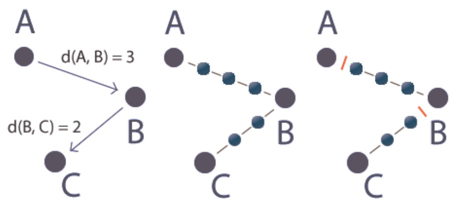

FIG. 1. Example of the construction of a Scheduled Networks.

A wide set of measures can be defined based on such network structure: and many of them are the result of a transformation from a distance-based to a time-based domain. Moreover, some dynamical aspects of the system can be described with ad-hoc calculations: for example, the number of alternative paths available connecting two nodes in a given time window, the mean cost (expressed in time) to go from one node to another, or the centrality of a node, measured as the increase of the mean travel cost if such node is deleted from the network. Of special interest in transportation networks is the study of uncertainty: by varying the scheduling information it is possible to simulate delays or jams, and see how those events affect the travel cost.

While we have focused in transportation networks as a guide example, the formalism we propose can indeed be applied to many other contexts. For example, biological networks, where some interactions can occur only in fixed sequences; financial networks, resulting from tracking interactions between different firms in time; or business planning, where projects can be seen as a collections of operations, each one with a time duration and initial conditions, linked in some logical (and complex) sequences.

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